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ABJM with Flavors and FQHE

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Abstract

We add fundamental matters to the $\mathcal{N} = 6$ Chern-Simons theory (ABJM theory), and show that D6-branes wrapped over $AdS_4 \times S^3/\mathbb{Z}_2$ in type IIA superstring theory on $AdS_4 \times \mathbb{CP}^3$ give its dual description with $\mathcal{N} = 3$ supersymmetry. We confirm this by the arguments based on R-symmetry, supersymmetry, and brane configuration of ABJM theory. We also analyze the fluctuations of the D6-brane and compute the conformal dimensions of dual operators. In the presence of fractional branes, the ABJM theory can model the fractional quantum Hall effect (FQHE), with RR-fields regarded as the external electric-magnetic field. We show that an addition of the flavor D6-brane describes a class of fractional quantum Hall plateau transitions.

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1 Introduction

Recently, it was pointed out in [1] that the three-dimensional $\mathcal{N} = 6$ Chern-Simons theory with $U(N)_k \times U(N)_{-k}$ (ABJM theory)[1] has a holographic dual description in terms of M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ or type IIA superstring theory on $AdS_4 \times \mathbb{CP}^3$. After the discovery, there has been remarkable progress in understanding AdS/CFT correspondence [2] in three dimensions, i.e., AdS_4/CFT_3 correspondence. Even in that situation, this duality is not yet understood at the level of more familiar AdS_5/CFT_4 correspondence. One of the important aspects is how to add flavors to the AdS_4/CFT_3 correspondence. In the ABJM theory we can add flavor fields which belong to the fundamental representation of the gauge group. Therefore the problem is how to reproduce the same setup in the holographic dual theory.

This is important not only from the viewpoint of holographic duality but also for the purpose of the application to some realistic models in condensed matter physics. Indeed, the ABJM theory has been employed to realize fractional quantum Hall effect (FQHE) recently in [3] by considering edge states.[2] Notice also that in the standard Chern-Simons Ginzburg-Landau description of FQHE, we need a charged scalar field. Such a field cannot be found in the ABJM theory unless we introduce other fields such as flavors.

The main purpose of this paper is to find a holographic description of the $\mathcal{N} = 3$ flavors in ABJM theory in terms of type IIA superstring theory, motivated by a connection to FQHE. We will concentrate on the case with a few number of flavors, therefore the probe approximation is valid for large N of gauge group $U(N) \times U(N)$. This is also the case for describing flavors in $\mathcal{N} = 4$ super Yang-Mills gauge theory by adding probe D7-branes to $AdS_5 \times S^5$ [5]. We find that the flavor would be introduced if we consider D6-branes wrapped over the Lens space $S^3/\mathbb{Z}_2$ (or equivalently the real projective space $\mathbb{RP}^3$) in $\mathbb{CP}^3$. This D6-brane has the degrees of freedom of choosing a $\mathbb{Z}_2$ Wilson line and we will identify this possibility with the choice of two gauge fields of ABJM theory to which we add flavors. The probe D6-brane respects desired R-symmetry and supersymmetry, and the same conclusion can be derived from the argument based on the brane construction of ABJM theory [1]. We will also calculate the fluctuations of the D6-branes and observe that the conformal dimensions of dual operators obtained from the analysis are consistent with what are expected from the gauge theory side. In addition, we will give a simple realization of FQHE in AdS_4/CFT_3 by adding the fractional D2-branes to the ABJM theory. Adding flavor D6-branes to this setup, we will give a realization of fractional quantum Hall plateau transitions.

The organization of this paper is as follows; In section 2, we will show how to introduce flavors to the ABJM theory and which D6-brane embedding corresponds to the flavors. This is based on the analysis of R-symmetry, supersymmetry and tyep IIB brane construction of ABJM theory. In section 3, we perform the analysis of fluctuation spectrum of the D6-branes. We will

[1] Here k and $-k$ denote the levels of Chern-Simons theory with each $U(N)$.

[2] See also [4] for other possibilities of realization of FQHE.

find the conformal dimensions of the dual operators and confirm our identification of the flavor D6-branes. In section 4, we will show how to model FQHE systems by adding the fractional D2-branes to ABJM theory. Moreover, we will realize fractional quantum Hall plateau transitions in the presence of the flavor D6-brane. Section 5 reviews our conclusions and suggests possibilities for future work.

2 Flavor D6-branes in ABJM Theory

The $\mathcal{N}=6$ supersymmetric Chern-Simons gauge theory in three dimensions, often called ABJM theory, was shown to be dual to type IIA superstring on $AdS_4 \times \mathbb{CP}^3$ by taking the large N scaling limit with $\lambda = \frac{N}{k}$ kept finite [1]. This theory has the gauge group $U(N) \times U(N)$ with the level k and $-k$, respectively. One of the most important deformations of this theory should be adding flavors which belong to the fundamental representation of the gauge groups. Since there are two gauge groups, we expect two kinds of flavors. In the field theory side it is straightforward to construct such a theory, and the purpose of this section is to identify which configuration is the holographic dual of the theory with flavors. We work within the probe brane approximation, therefore the fundamental flavors are quenched. We concentrate on the case preserving the maximal supersymmetry, that is, the theory should possess $\mathcal{N}=3$ supersymmetry.

2.1 ABJM Theory with Flavors

First let us include flavors in the ABJM theory from the viewpoint of gauge theory. The ABJM theory consists of two gauge multiplets for the two copies of the gauge groups $U(N) \times U(N)$ and bi-fundamental chiral fields (A_1, A_2) in the $(N, \bar{N})$ representation and (B_1, B_2) in the $(\bar{N}, N)$ representation. In order to add flavors to the ABJM theory, we would like to introduce hypermultiplets while keeping $\mathcal{N}=3$ supersymmetry. To achieve this, we add either of or both of the chiral multiplets $(Q_1, \tilde{Q}_1)$ and $(Q_2, \tilde{Q}_2)$. Here chiral superfields Q_1 and Q_2 belong to $(N, \mathbf{1})$ and $(\mathbf{1}, N)$ representation, respectively; and chiral superfields $\tilde{Q}_1$ and $\tilde{Q}_2$ belong to $(\bar{N}, \mathbf{1})$ and $(\mathbf{1}, \bar{N})$, respectively. For general discussions of the $\mathcal{N}=3$ Chern-Simons theory, refer to e.g. [6, 1].

In the ABJM theory, the interaction is essentially described by the superpotential

$$W_{ABJM} = \mathrm{Tr}[\varphi_1^2 - \varphi_2^2] + \mathrm{Tr} B_i \varphi_1 A_i + \mathrm{Tr} A_i \varphi_2 B_i \ , \tag{2.1}$$

where φ_1 and φ_2 are the chiral superfields in the gauge multiplets. We have added hypermultiplets to the ABJM theory, but the interaction is determined by the requirement of $\mathcal{N}=3$ supersymmetry and it is given by the following superpotential

$$W_{flavor} = \mathrm{Tr} \tilde{Q}_1 \varphi_1 Q_1 + \mathrm{Tr} \tilde{Q}_2 \varphi_2 Q_2 \ . \tag{2.2}$$

Originally we have $SU(4)$ R-symmetry which rotates $(A_1, A_2, \bar{B}_1, \bar{B}_2)$ in the ABJM theory. Even after the flavors are added the theory still preserves $\mathcal{N} = 3$ supersymmetry and the R-symmetry is now $SU(2)$, which acts on the doublet $(A_i, \bar{B}_i)$ and $(Q_i, \tilde{Q}_i)$. In addition, this $\mathcal{N} = 3$ supersymmetric theory have an extra internal $SU(2)$ symmetry which acts on the doublets (A_1, A_2) and (B_1, B_2) simultaneously. Therefore the theory has the symmetry $SU(2)_R \times SU(2)_I$.

In this way we have shown that one or two kinds of flavors $(Q_1, \tilde{Q}_1)$ and $(Q_2, \tilde{Q}_2)$ can be introduced while preserving $\mathcal{N} = 3$ supersymmetry. Notice that mesonic operator can be constructed as $\tilde{Q}_1(AB)^l Q_1$ or $\tilde{Q}_1(AB)^l AQ_2$. If there is only one type of hypermultiplets, then we have only the former type of mesonic operator. If there are both flavors, we will have both operators. In the rest of this section we will see how they are realized by adding flavor D6-branes in the holographic dual geometry. In particular, the brane configuration realizing the ABJM theory with flavors is constructed and the duality map confirms the proposal.

2.2 $AdS_4 \times \mathbb{CP}^3$ Geometry

It might be useful to start from the geometry dual to the ABJM theory in order to identify the relevant D6-brane embedding. It was argued in [1] that the dual theory is type IIA superstring on $AdS_4 \times \mathbb{CP}^3$, whose metric is given by[3]

$$ds^2 = L^2(ds^2_{AdS_4} + 4ds^2_{\mathbb{CP}^3}) , \tag{2.3}$$

where $L^2 = R^3/(4k)$ with $R^6 = 2^5\pi^2 Nk$. The dilaton field is $e^{2\phi} = R^3/k^3 = 2^{\frac{5}{2}}\pi\sqrt{N/k^5}$ and the background fluxes are

$$F_2 = \frac{2k^2}{R^3}\omega , \quad \tilde{F}_4(\equiv F_4 - C_1 \wedge H_3) = -\frac{3}{8}R^3\epsilon_{AdS_4} , \quad H_3 = 0 . \tag{2.4}$$

Here ϵ_{AdS_4} is the volume form of the unit radius AdS_4 and ω is the Kähler form of $\mathbb{CP}^3$. The metric of $\mathbb{CP}^3$ can be written down explicitly as in (2.9), but it is instructive to construct the metric for later purpose. This background preserves 24 out of total 32 supersymmetries of type IIA supergravity [1, 8].

The metric of $\mathbb{CP}^3$ can be obtained by taking large k limit of the orbifold $S^7/\mathbb{Z}_k$. Actually this fact was used to construct the dual geometry (2.3) through the dimensional reduction of the near horizon geometry of M2-branes at the orbifold $\mathbb{C}^4/\mathbb{Z}_k$, which is given by $AdS_4 \times S^7/\mathbb{Z}_k$. We can express S^7 by the complex coordinates X_1, X_2, X_3 and X_4 with the constraint $|X_1|^2 + |X_2|^2 + |X_3|^2 + |X_4|^2 = 1$. It is convenient to parameterize S^7 as[4]

$$\begin{aligned} X_1 &= \cos\xi\,\cos\frac{\theta_1}{2}\,e^{i\frac{\chi_1+\varphi_1}{2}} , \qquad & X_2 &= \cos\xi\,\sin\frac{\theta_1}{2}\,e^{i\frac{\chi_1-\varphi_1}{2}} , \\ X_3 &= \sin\xi\,\cos\frac{\theta_2}{2}\,e^{i\frac{\chi_2+\varphi_2}{2}} , \qquad & X_4 &= \sin\xi\,\sin\frac{\theta_2}{2}\,e^{i\frac{\chi_2-\varphi_2}{2}} , \end{aligned} \tag{2.5}$$

[3] In this paper we assume $\alpha' = 1$ and follow the notations in [7].
[4] We follow the notation in [9].

where the ranges of the angular variables are $0 \le \xi < \frac{\pi}{2}$, $0 \le \chi_i < 4\pi$, $0 \le \varphi_i < 2\pi$ and $0 \le \theta_i < \pi$. The $\mathbb{Z}_k$ orbifold action is taken along the y-direction as $y \sim y + \frac{2\pi}{k}$, where the new coordinate y is defined by

$$\chi_1 = 2y + \psi \ , \qquad \chi_2 = 2y - \psi \ . \tag{2.6}$$

In the new coordinate system, the S^7 can be rewritten as

$$ds^2_{S^7} = ds^2_{\mathbb{CP}^3} + (dy + A)^2, \tag{2.7}$$

where

$$A = \frac{1}{2}(\cos^2\xi - \sin^2\xi)d\psi + \frac{1}{2}\cos^2\xi\cos\theta_1 d\varphi_1 + \frac{1}{2}\sin^2\xi\cos\theta_2 d\varphi_2 \ . \tag{2.8}$$

In this way we find the metric of $\mathbb{CP}^3$ as

$$\begin{aligned} ds^2_{\mathbb{CP}^3} = d\xi^2 + \cos\xi^2\sin^2\xi\left(d\psi + \frac{\cos\theta_1}{2}d\varphi_1 - \frac{\cos\theta_2}{2}d\varphi_2\right)^2 \\ + \frac{1}{4}\cos^2\xi\left(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2\right) + \frac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2) \ . \end{aligned} \tag{2.9}$$

The ranges of the angular valuables are given by

$$0 \le \xi < \frac{\pi}{2} \ , \quad 0 \le \psi < 2\pi \ , \quad 0 \le \theta_i < \pi \ , \quad 0 \le \varphi_i \le 2\pi \ . \tag{2.10}$$

In this coordinate system, the RR 2-form $F_2 = dC_1$ in the type IIA string is explicitly given by

$$\begin{aligned} F_2 \quad &= \quad k\Big(-\cos\xi\sin\xi d\xi\wedge(2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2) \\ &\quad -\frac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1\wedge d\varphi_1 - \frac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2\wedge d\varphi_2\Big) \equiv -\frac{2k^2}{R^3}\omega \ . \end{aligned} \tag{2.11}$$

The explicit expression of the Kähler form ω can also be read off from this equation.

2.3 Flavor D6-branes

Utilizing the explicit metric of $\mathbb{CP}^3$, we would like to discuss the D-brane configuration in $AdS_4 \times \mathbb{CP}^3$, which is dual to adding flavors to ABJM theory. The corresponding D6-brane should be wrapped over AdS_4 times a topologically trivial[5] 3-cycle in $\mathbb{CP}^3$. As discussed above, the original ABJM theory has $SU(4)$ R-symmetry, which corresponds to the $SU(4)$ symmetry of $\mathbb{CP}^3$. Adding the flavors reduces the R-symmetry into $SO(4) = SU(2)_R \times SU(2)_I$, therefore we should find a cycle with the $SO(4)$ symmetry.

[5]If a brane is wrapped over a topologically trivial cycle in $\mathbb{CP}^3$, then the brane over AdS_4 does not carry any charge. Otherwise, it is not possible to wrap the whole AdS_4 space. The topologically trivial cycle tends to shrink due to the brane tension, but this brane configuration can be actually stabilized due to the curvature of AdS space. See, e.g., [5] for more detail.

We assume that the coordinates (X_1, X_2, X_3, X_4) correspond to $(A_1, \bar{B}_1, \bar{B}_2, A_2)$, and in this case the $SU(2)$ symmetry rotates (X_1, X_2) and (X_3, X_4) at the same time. We want to have a 3-cycle invariant under this rotation, and a natural one is given by

$$\theta_1 = \theta_2 (= \theta) \ , \quad \varphi_1 = -\varphi_2 (= \varphi), \quad \xi = \frac{\pi}{4} \ . \tag{2.12}$$

The induced metric becomes

$$ds^2 = 4L^2 \left(\frac{1}{4}(d\psi + \cos\theta d\varphi)^2 + \frac{1}{4}(d\theta^2 + \sin^2\theta d\varphi^2) \right) \ , \tag{2.13}$$

where $0 \leq \psi < 2\pi$, $0 \leq \theta < \pi$ and $0 \leq \varphi < 2\pi$. This metric looks like the metric of a regular S^3 with the unit radius,[6] though in that case the periodicity should be $0 \leq \psi < 4\pi$ instead of $0 \leq \psi < 2\pi$. Therefore we conclude that the 3-cycle we found is actually the Lens space $S^3/\mathbb{Z}_2$. If we describe the S^3 by $w_1^2 + w_2^2 + w_3^2 + w_4^2 = 1$, then the $\mathbb{Z}_2$ orbifold action is given by

$$(w_1, w_2, w_3, w_4) \to (-w_1, -w_2, -w_3, -w_4) \ . \tag{2.14}$$

An important property is that this $\mathbb{Z}_2$ action is the center of $SO(4)$, and hence a D6-brane wrapped over this $S^3/\mathbb{Z}_2$ preserves the $SO(4)$ symmetry as expected. The volume of $S^3/\mathbb{Z}_2$ can be computed as $\mathrm{Vol}(S^3/\mathbb{Z}_2) = 8\pi^2 L^3$. As we will see later, this $S^3/\mathbb{Z}_2$ is the same as the $\mathbb{RP}^3$ which is embedded into $\mathbb{CP}^3$ in a rather trivial way.

It is important to notice that there is a non-trivial torsion cohomology as

$$H^2(S^3/\mathbb{Z}_2, \mathbb{Z}) = \mathbb{Z}_2 \ , \tag{2.15}$$

and a gauge theory on this manifold has two vacua due to the $\mathbb{Z}_2$ torsion. Let us define $[\alpha]$ as the torsion 1-cycle in $S^3/\mathbb{Z}_2$ generated by $0 \leq \psi < 2\pi$, then the $\mathbb{Z}_2$ charge is interpreted as the $\mathbb{Z}_2$ Wilson loop

$$e^{i\int_{[\alpha]} A} = \pm 1 \ . \tag{2.16}$$

In other words, we can construct two types of D6-branes depending on the Wilson loop. In the following, we will show that one of them provides a flavor for one of the two $U(N)$ gauge groups and the other does the other one. This is motivated by the Douglas-Moore prescription of D-branes at $\mathbb{Z}_2$ orbifold [10], although our setup is a T-dual of them.

2.4 Supersymmetry of D6-brane

In the previous subsection we have found a candidate 3-cycle on which the flavor D6-brane should be wrapped based on the R-symmetry argument. Since a three-dimensional $\mathcal{N} = 3$ superconformal field theory possesses 12 supersymmetries, our flavor brane configuration should

[6] Notice that we can rewrite (2.13) as $ds^2 = d\xi^2 + \cos^2\xi d\phi_1^2 + \sin^2\xi d\phi_2^2$ by setting $\xi = \theta/2$, $\psi = \phi_1 + \phi_2$ and $\varphi = \phi_1 - \phi_2$.

preserve half of the 24 supersymmetries in the bulk. In order to count the number of supersymmetries of the D6-brane configuration in $AdS_4 \times \mathbb{CP}^3$, it is useful to uplift to M-theory and to utilize the Killing spinor in $AdS_4 \times S^7/\mathbb{Z}_k$. Adding the eleventh dimension y, the two ten-dimensional 16-components (Weyl) Killing spinors $\{\epsilon_\pm\}$ is combined into a 11D 32-component Killing spinor. Following [11] we define the angular coordinates $X_i = \mu_i e^{i\zeta_i}$ instead of (2.5) with $\{\mu_1, \mu_2, \mu_3, \mu_4\} = \{\sin\alpha, \cos\alpha\sin\beta, \cos\alpha\cos\beta\sin\gamma, \cos\alpha\cos\beta\cos\gamma\}$. The Killing spinor is now given by

$$\epsilon = e^{\frac{\alpha}{2}\hat{\gamma}\gamma_4} e^{\frac{\beta}{2}\hat{\gamma}\gamma_5} e^{\frac{\gamma}{2}\hat{\gamma}\gamma_6} e^{\frac{\xi_1}{2}\gamma_{47}} e^{\frac{\xi_2}{2}\gamma_{58}} e^{\frac{\xi_3}{2}\gamma_{69}} e^{\frac{\xi_4}{2}\hat{\gamma}\gamma_{10}} e^{\frac{\rho}{2}\hat{\gamma}\gamma_1} e^{\frac{t}{2}\hat{\gamma}\gamma_0} e^{\frac{\theta}{2}\gamma_{12}} e^{\frac{\phi}{2}\gamma_{23}} \epsilon_0 \ , \tag{2.17}$$

where $(x^0, x^1, \cdots, x^{10}) = (t, r, \theta, \phi, \alpha, \beta, \gamma, \xi_1, \xi_2, \xi_3, \xi_4)$ and ϵ_0 is a constant 32-component Majorana spinor in 11D. The eleventh dimension y is a linear combination of the four phases $\{\zeta_i\}$.

In $AdS_4 \times \mathbb{CP}^3$, consider a D6-brane extending along the entire AdS_4 and the $\{\alpha, \beta, \gamma\}$-directions while sitting at constant phase directions. When lifted to M-theory, it corresponds to a Taub-NUT spacetime along the 016789-directions. Then the supersymmetries preserved are given by the constraint

$$\Gamma_6 \epsilon = \epsilon \qquad \text{where} \qquad \Gamma_6 = \gamma_{0123456} \ . \tag{2.18}$$

Therefore it projects out half of the supersymmetries by

$$\gamma_{0123456}\epsilon_0 = \epsilon_0 \ . \tag{2.19}$$

Then the orbifolding action $z_i \to z_i e^{i2\pi/k}$ further projects out 4 supersymmetries. In total, this 11D system with the Taub-NUT spacetime has 12 supersymmetries. Performing the dimensional reduction on the y-direction, we return to the D6-brane extending along AdS_4 and the $\{\alpha, \beta, \gamma\}$-directions inside $\mathbb{CP}^3$ and it preserves 12 supersymmetries. Since $\{\alpha, \beta, \gamma\}$ are the three real directions in $\mathbb{CP}^3$, the 3-cycle wrapped by the D6-brane is $\mathbb{RP}^3$.

Utilizing the $SU(4)$ symmetry of $\mathbb{CP}^3$, we can show that the above D6-brane configuration is indeed the one obtained before. We perform the following $SU(4)$ symmetry transformation of $\mathbb{CP}^3$

$$\frac{1}{\sqrt{2}}(X_1 + X_3) \to X_1 \ , \quad \frac{-i}{\sqrt{2}}(X_1 - X_3) \to X_2 \ , \quad \frac{1}{\sqrt{2}}(X_2 + X_4) \to X_3 \ , \quad \frac{i}{\sqrt{2}}(X_2 - X_4) \to X_4 \ . \tag{2.20}$$

Then, the cycle defined by the condition (2.12) is mapped to the one with the induced metric

$$ds^2 = d\xi^2 + \frac{1}{4}\cos^2\xi d\theta_1^2 + \frac{1}{4}\sin^2\xi d\theta_2^2 \ , \tag{2.21}$$

which may be given by the replacement

$$\psi + \varphi \to \theta_1 \ , \quad -\psi + \varphi \to \theta_2 \ , \quad \frac{\theta}{2} \to \xi \ . \tag{2.22}$$

This cycle can also be obtained by setting $\varphi_i = 0$ and $\chi_i = 0$ in the new coordinates (2.5). If we take into account the presence of $\mathbb{Z}_k$ orbifold action carefully, we can find that the ranges

of coordinates are $0 \leq \xi < \pi/2$ and $0 \leq \theta_i < 2\pi$ with the $\mathbb{Z}_2$ identification $\theta_1 \to \theta_1 + \pi$ and $\theta_2 \to \theta_2 + \pi$. Thus we again obtain $S^3/\mathbb{Z}_2$, which can be mapped to the previous one (2.13) by the $SU(4)$ symmetry of the background. In this way we have proved that the D6-brane over $S^3/\mathbb{Z}_2$ discussed in the previous subsection preserves 12 supersymmetries as we wanted to show. Notice also that from this construction we can clearly understand the 3-cycle $S^3/\mathbb{Z}_2$ as the $\mathbb{RP}^3$ inside $\mathbb{CP}^3$.

2.5 Relation to Brane Configuration

One of the confirmation of the duality between the ABJM theory and type IIA superstring on $AdS_4 \times \mathbb{CP}^3$ is made through the realization of ABJM theory with the type IIB brane configuration [1] (see also [12, 13]). Therefore, the construction of brane configuration corresponding to the ABJM theory with flavors would give a strong support of our identification of flavor D-brane.

Let us begin with the ABJM theory without flavor. We introduce a standard cartesian coordinate $x^0, x^1, \cdots, x^9$ with x^6 compactified on a small circle. Then the type IIB brane configuration of the ABJM theory is given by N D3-branes which extend in the 0126-directions, a NS5-brane in 012345 and a $(1,k)$5-brane in $012[3,7]_\theta[4,8]_\theta[5,9]_\theta$. Here $[i,j]_\theta$ means that it extends in the particular direction between ∂_i and ∂_j so that it preserves $\mathcal{N}=3$ supersymmetry [12, 13]. Since the NS5-brane and $(1,k)$5-brane divide the circular D3-branes into two segments, the gauge group becomes $U(N) \times U(N)$. The chiral matter multiplets A_i and B_i come from open strings between these two parts of the D3-branes.

In order to introduce flavors to ABJM theory, we need to insert D5-branes to this setup. Here we introduce D5-branes in the 012789-directions such that the brane configuration preserves $\mathcal{N}=3$ supersymmetry as confirmed in appendix A.[7] Since there are two segments of D3-branes, we can insert D5-branes in either or both of these two segments. If we insert a D5-brane in a segment, then we have one type of hypermultiplets, say, $(Q_1, \tilde{Q}_1)$. If we insert another D5-brane in the other segment, then we have one more type of hypermultiplets $(Q_2, \tilde{Q}_2)$. When N_1^f and N_2^f D5-branes are inserted in each segments of D3-branes, the number of flavors for Q_1 and Q_2 are increased by N_1^f and N_2^f, respectively. In this paper we will set N_1^f and N_2^f to be zero or one. Mesonic operators may be interpreted as the strings stretching between the D5-branes, and strings between the same brane correspond to the type of $\tilde{Q}_1(AB)^l Q_1$ and strings between the different branes correspond to the type of $\tilde{Q}_1(AB)^l AQ_2$. Notice that this D5-brane is a standard flavor D-brane in the brane configurations of three-dimensional $\mathcal{N}=4$ supersymmetric Yang-Mills gauge theory [15].

In the following we will show that when mapped to type IIA theory the above D5-brane actually corresponds to the D6-brane wrapped over AdS_4 times the cycle $S^3/\mathbb{Z}_2$ (2.13) obtained

[7] The same D5-brane is also discussed in the IIB brane configuration in the independent work [14], quite recently from a different motivation.

above. We start with the case without flavor again. Via the standard duality map, the type IIB brane configuration can be lifted to M-theory with M2-branes at the intersection of two KK monopoles. Before adding the M2-branes, this geometry takes the form of $R^{1,2} \times X_8$ and the explicit metric of X_8 can be found in [1] (see also [16]). There the coordinates of eight-dimensional manifold X_8 were expressed by $(\varphi_1, \vec{x}^1)$, $(\varphi_2, \vec{x}^2)$, which is essentially two copies of Taub-NUT spacetimes warped with each other. The relation between this coordinate of X_8 and the brane configuration is given by $\varphi_1 = x^6$, $\varphi_2 = x^{10}$, $\vec{x}^1 = (x^7, x^8, x^9)$ and $\vec{x}^2 = (x^3, x^4, x^5)$. Comparing with the coordinates (X^1, X^2, X^3, X^4) of $\mathbb{C}^4/\mathbb{Z}_k$ in (2.5), we have

$$\begin{aligned}
&\chi_1 = -2\varphi_1' \equiv -2(\varphi_1 - \frac{\varphi_2}{k}) \ , \quad \chi_2 = \varphi_2' \equiv \frac{\varphi_2}{k} \ , \\
&\vec{x}_1' = \vec{x}_1 = r^2 \cos^2\xi(\cos\theta_1, \sin\theta_1\cos\varphi_1, \sin\theta_1\sin\varphi_1) \ , \\
&\vec{x}_2' = \vec{x}_1 + k\vec{x}_2 = r^2 \sin^2\xi(\cos\theta_2, \sin\theta_2\cos\varphi_2, \sin\theta_2\sin\varphi_2) \ ,
\end{aligned} \tag{2.23}$$

where r is defined by $\sum_{i=1}^4 |X_i|^2 = r^2$. This leads to $x^6 = \psi$ and $x^{10} = ky - \frac{k}{2}\psi$.

We would like to introduce a 6-brane in this setup. Our D6-brane wrapped over $S^3/\mathbb{Z}_2$ (2.13) corresponds to a KK-monopole in M-theory. As is clear from the description in the coordinate (2.5), it is simply expressed as the codimension-three surface of $\vec{x}_1' = \vec{x}_2'$, which leads to the constraint $\vec{x}_2 = 0$. Taking the T-duality in the 6-direction (notice that the D6-brane extends in the 6-direction), it becomes a D5-brane in the IIB string which extends in the 012789-directions. This argument almost confirmed that our D6-brane over $S^3/\mathbb{Z}_2$ corresponds to the D5-brane introduced in the type IIB brane configuration with one subtlety. Namely, we only have to explain the fact that a D5-brane can be inserted in either of the two segments. Actually this fact is consistent with our D6-brane setup since we have the choice of $\mathbb{Z}_2$ Wilson loop in the ψ-direction. Performing the T-duality in the $x^6 = \psi$ direction, these two possibilities correspond to the two segments of the D3-branes on which we can place the D5-brane.

3 Meson Spectrum from Flavor Brane

One of the most important checks of AdS/CFT correspondence is the comparison of spectrum. In this section we would like to investigate the fluctuation of D6-brane wrapping over $AdS_4 \times S^3/\mathbb{Z}_2$ inside $AdS_4 \times \mathbb{CP}^3$. The spectrum of the fluctuation should be reproduced by the conformal dimensions of dual operators. We will study the fluctuation of a scalar mode transverse to the $S^3/\mathbb{Z}_2$ in $\mathbb{CP}^3$ and the gauge field on the worldvolume. We start from the D6-brane action

$$\begin{aligned}
S_{D6} = &-\frac{1}{(2\pi)^6}\int d^{1+6}x e^{-\phi}\sqrt{-\det(g_{ab} + 2\pi F_{ab})} \\
&+ \frac{(2\pi)^2}{2(2\pi)^6}\int C_3 \wedge F \wedge F + \frac{1}{(2\pi)^6}\int C_7 \ .
\end{aligned} \tag{3.1}$$

Here g_{ab} is the induced metric of the D6-brane, F_{ab} is the field strength on the worldvolume, and C_3, C_7 are the induced 3-form and 7-form potentials. There are other types of Chern-Simons term, but we included only those relevant for our purpose. We adopt the static gauge and the measure is $d^{1+6}x = dtdxdydrd\theta d\psi d\varphi$. In the following we use μ for t, x, y and i, j for S^3 coordinates. In the i, j label directions are given by

$$d\sigma^1 = \frac{1}{2}d\theta \ , \qquad d\sigma^2 = \frac{1}{2}d\psi \ , \qquad d\sigma^3 = \frac{1}{2}\sin\theta d\varphi \ . \tag{3.2}$$

In the case of D7-brane in $AdS_5 \times S^5$ similar analyses have been done in [17] (see also [5]).

3.1 Scalar perturbation

First we study the fluctuations of a scalar mode, which correspond to the scalar perturbation of D6-brane orthogonal to the worldvolume directions. Here we consider only the fluctuation of $\xi = \pi/4 + \eta$ with small η and the fluctuations along the other two directions will be obtained by the symmetry argument. For this purpose the Chern-Simons term with 7-form potential is important. Using the fact that $F_2 = *F_8 = -\frac{2k^2}{R^3}\omega$, the 7-form potential C_7 can be written as

$$C_7 = -\frac{k^2 L^4}{R^3}\sigma \wedge \omega \wedge r^2 dt \wedge dx \wedge dy \wedge dr \ , \tag{3.3}$$

where σ is defined by $d\sigma = \omega$. Under the condition of $\theta_1 = \theta_2$ and $\varphi_1 + \varphi_2 = 0$, we find

$$\begin{aligned} \sigma &= -L^2(\cos^2\xi - \sin^2\xi)(d\psi + \cos\theta d\varphi) \ , \\ \omega &= L^2\left(4\cos\xi\sin\xi d\xi \wedge d\psi + (\cos^2\xi - \sin^2\xi)\sin\theta d\theta \wedge d\varphi + 4\cos\xi\sin\xi\cos\theta d\xi \wedge d\varphi\right) \ , \end{aligned} \tag{3.4}$$

therefore we have

$$C_7 = \frac{k^2 L^8}{R^3}(\cos^2\xi - \sin^2\xi)^2 r^2 \sin\theta dt \wedge dx \wedge dy \wedge dr \wedge d\psi \wedge d\theta \wedge d\varphi \ . \tag{3.5}$$

Expanding the D6-brane action (3.1), the quadratic term of η is given by

$$\delta S = \frac{k}{2(2\pi)^6 L}\int d^{1+6}x\sqrt{-\det g}(2g^{ab}\partial_a\eta\partial_b\eta - 4\eta^2) \ . \tag{3.6}$$

In our notation $\sqrt{-\det g} = L^7 r^2 \sin\theta$. The equation of motion for η leads to

$$\frac{1}{\sqrt{-\det g}}\partial_a(\sqrt{-\det g}g^{ab}\partial_b)\eta + 2\eta = 0 \ , \tag{3.7}$$

therefore we have

$$\frac{1}{r^2}\partial_\mu\partial^\mu\eta + \frac{1}{r^2}\partial_r(r^4\partial_r)\eta + \tfrac{1}{4}D_i D^i\eta + 2\eta = 0 \ . \tag{3.8}$$

Here D_i represents covariant derivatives on S^3. Using the separation of variables we can write as

$$\eta = \rho(r)e^{ik\cdot x}Y^l(S^3) \tag{3.9}$$

with the spherical harmonics

$$D_i D^i Y^l(S^3) = -l(l+2)Y^l(S^3) \ . \tag{3.10}$$

As discussed in the apppendix B, the $\mathbb{Z}_2$ orbifold action restricts $l \in 2\mathbb{Z}$. If the scalar field feels the $\mathbb{Z}_2$ holonomy along the ψ-direction, then the restriction is $l \in 2\mathbb{Z}+1$. With the help of separation of variables, the equation of motion reduces to

$$\left[\partial_r^2 + \frac{4}{r}\partial_r + \frac{8-l(l+2)}{4r^2} - \frac{k^2}{r^4}\right]\rho(r) = 0 \ . \tag{3.11}$$

We assume the regularity at the horizon of AdS_4, i.e. at $r=0$. Then the above equation can be solved by the modified Bessel function as

$$\rho(r) = r^{-\frac{3}{2}} K_{\frac{l+1}{2}}(\tfrac{k}{r}) \ . \tag{3.12}$$

As usual the conformal dimension of dual operator Δ can be read off from the boundary behavior at $r \to \infty$ as $r^{-\Delta}$ or $r^{3-\Delta}$. Expanding the solution around $r \to \infty$, we find

$$\rho(r) \sim c_1 r^{-\frac{3}{2}+\frac{l+1}{2}} + c_2 r^{-\frac{3}{2}-\frac{l+1}{2}} \ , \tag{3.13}$$

with some coefficients c_1, c_2. Therefore the conformal dimension of dual operator is

$$\Delta = \frac{l}{2} + 2 \ . \tag{3.14}$$

Without any holonomy the conformal dimension is $\Delta = 2+n$ with $n = 0,1,2,\cdots$ and the dual operator is of the form $\tilde{\psi}_1(AB)^n\psi_1$. The lowest one $n=0$ is interpreted as the (supersymmetric) mass deformation of the flavor. In the case with $\mathbb{Z}_2$ holonomy, the conformal dimension is $\Delta = 2+n+1/2$ with $n=0,1,2,\cdots$ and the dual operator is of the form $\tilde{\psi}_1(AB)^n A\psi_2$.[8]

3.2 Vector perturbation

On a D6-brane, there is a $U(1)$ gauge field and we can study the spectrum due to a small shift of the gauge field. The equations of motion are given by

$$\frac{1}{\sqrt{-\det g}}\partial_a(\sqrt{-\det g}F^{ab}) - \frac{3}{8}\epsilon^{bij}\partial_i A_j = 0 \ . \tag{3.15}$$

Here we have used the fact that the induced 3-form potential can be written as

$$C_3 = -\frac{kL^2}{2} r^3 dt \wedge dx \wedge dy \ . \tag{3.16}$$

[8]Since the other two scalar modes can be obtained by the symmetry transformation of $\mathbb{CP}^3$ while fixing the $S^3/\mathbb{Z}_2$, it is natural to guess that the conformal dimensions of their dual operators are also the same as in this case.

Following [17] it can be shown that it is enough to consider the following three types of gauge field configuration. They are given by

$$\text{Type I:} \quad A_\mu = 0 \ , \quad A_r = 0 \ , \quad A_i = \rho_I^\pm(r) e^{ik\cdot x} Y_i^{l,\pm}(S^3) \ , \tag{3.17}$$

$$\text{Type II:} \quad A_\mu = \xi_\mu \rho_{II}(r) e^{ik\cdot x} Y^l(S^3) \ , \quad \xi \cdot k = 0 \ , \quad A_r = 0 \ , \quad A_i = 0 \ , \tag{3.18}$$

$$\text{Type III:} \quad A_\mu = 0 \ , \quad A_r = \rho_{III}(r) e^{ik\cdot x} Y^l(S^3) \ , \quad A_i = \tilde{\rho}_{III}(r) e^{ik\cdot x} D_i Y^l(S^3) \ . \tag{3.19}$$

The vector components along S^3 directions can be expanded by the vector spherical harmonics, which satisfy

$$\begin{aligned} D_i D^i Y_j^{l,\pm} - R_j^k Y_k^{l,\pm} &= -(l+1)^2 Y_j^{l,\pm} \ , \\ \epsilon_{ijk} D_j Y_k^{l,\pm} &= \pm(l+1) Y_k^{l,\pm}, \quad D^i Y_i^{l,\pm} = 0 \ , \end{aligned} \tag{3.20}$$

where $R_j^k = 2\delta_j^k$ is the Ricci tensor for S^3 with the unit radius. They belong to $(\frac{l\mp1}{2}, \frac{l\pm1}{2})$ representation with respect to $SU(2)_R \times SU(2)_L$.

Let us start with type I case. The equations of motion (3.15) lead to

$$\frac{1}{r^2}\partial_\mu \partial^\mu A^j + \frac{1}{r^2}\partial_r(r^4 \partial_r A^j) + \frac{1}{4}(D_i D^i A^j - 2\delta_i^j A^i) - \frac{3}{2}\epsilon_{jkl}\partial_k A^l = 0 \ . \tag{3.21}$$

Here we have used

$$\frac{1}{\sqrt{g}}\partial_i \sqrt{g}(\partial^i A^j - \partial^j A^i) = D_i D^i A^j + D^j D_i A^i - [D_i, D^j] A^i \tag{3.22}$$

and $D_i A^i = 0$, $[D_i, D^j] A^i = R_i^{\ j} A^i$. Then we find

$$\partial_r^2 \rho_I^\pm + 4\partial_r \rho_I^\pm - \frac{k^2}{r^4}\rho_I^\pm - \frac{(l+1)^2}{4r^2}\rho_I^\pm \mp \frac{6}{4}(l+1)\rho_I^\pm = 0 \ . \tag{3.23}$$

The solution for $\rho_I^+(r)$ regular at $r = 0$ is

$$\rho_I^+(r) = r^{-\frac{3}{2}} K_{\frac{l+4}{2}}(\tfrac{k}{r}) \sim c_1 r^{-\frac{1}{2}(l+7)} + c_2 r^{\frac{1}{2}(l+1)} \ , \tag{3.24}$$

thus the conformal weight of the dual operator is $\Delta_+ = \frac{l}{2} + \frac{7}{2}$, where $l \in 2\mathbb{Z}+1$ without holonomy and $l \in 2\mathbb{Z}$ with $\mathbb{Z}_2$ holonomy. The solution for $\rho_I^-(r)$ regular at $r = 0$ is

$$\rho_I^-(r) = r^{-\frac{3}{2}} K_{\frac{l-2}{2}}(\tfrac{k}{r}) \sim c_1 r^{-\frac{1}{2}(l+1)} + c_2 r^{\frac{1}{2}(l-5)} \ , \tag{3.25}$$

thus the conformal weight of the dual operator is $\Delta_- = \frac{l}{2} + \frac{1}{2}$ with the same condition for l as in ρ_I^+ case. The lowest one is given by $l = 1$ case, which is in the $(1, 0)$ representation and transforms as the triplet of $SU(2)_R$. The dual operator can be identified with the triplet $\mathcal{O}^1 = \{\bar{Q}Q - \bar{\tilde{Q}}\tilde{Q}, \tilde{Q}Q, \bar{\tilde{Q}}\bar{Q}\}$ of the scalar field in the hypermultiplet. This identification is quite important since the other cases follow only with the (super)symmetry arguments.

The type II case can be analyzed in the same way. The equations of motion lead to

$$\frac{1}{r^4}\partial_\nu\partial^\nu A_\mu + \partial_r\left(r^2\partial_r\left(\frac{1}{r^2}A_\mu\right)\right) + \frac{1}{4r^2}D_iD^iA_\mu = 0\ , \tag{3.26}$$

thus we obtain

$$\left(-\frac{k^2}{r^4} + \partial_r^2 + \frac{2}{r}\partial_r + \frac{8 - l(l+2)}{4r^2}\right)\rho_{II}(r) = 0\ . \tag{3.27}$$

The solution to this equation regular at $r = 0$ is given by

$$\rho_{II}(r) = r^{-\frac{3}{2}}K_{\frac{l+1}{2}}(\tfrac{k}{r}) \sim c_1 r^{\frac{l}{2}-1} + c_2 r^{-\frac{l}{2}-2}\ , \tag{3.28}$$

and the conformal dimension of the dual operator is $\Delta = \frac{l}{2} + 2$. The restriction to l is the same as the scalar case and $l \in 2\mathbb{Z}$ without holonomy and $l \in 2\mathbb{Z}+1$ with $\mathbb{Z}_2$ holonomy.

For type III case we first set $b = \mu$. Then we obtain the relation

$$\partial_r(r^2\rho_{III}(r)) - \tfrac{1}{4}l(l+2)\tilde{\rho}_{III}(r) = 0\ . \tag{3.29}$$

For $l = 0$, the solution behaves as $\rho_{III} \sim 1/r$. Since it is singular at $r = 0$ we remove $l = 0$ mode. Then the equations of motion for $b = r$ or $b = j$ read

$$r^2\partial_r^2(r^2\rho_{III}(r)) - k^2\rho_{III}(r) - \tfrac{1}{4}r^2 l(l+2)\rho_{III}(r) = 0\ , \tag{3.30}$$

and the solution regular at $r = 0$ is

$$\rho_{III}(x) = r^{-\frac{3}{2}}K_{\frac{l+1}{2}}(\tfrac{k}{r}) \sim c_1 r^{-\frac{l}{2}-2} + c_2 r^{\frac{l}{2}-1}\ , \tag{3.31}$$

thus $\Delta = \frac{l}{2} + 2$, where $l = 2, 4, 6, \cdots$ without holonomy and $l = 1, 3, 5, \cdots$ with $\mathbb{Z}_2$ holonomy.

3.3 Spectrum and the $\mathbb{Z}_2$ Wilson Loop

Let us summarize the results obtained in this section. Due to the choice of the $\mathbb{Z}_2$ Wilson loop, we have two types of D6-branes. Irrespective of the choice of Wilson loop, open strings on the same brane do not feel the effects of Wilson loop. Therefore, the scalar fields and the gauge field from the open string do not receive the $\mathbb{Z}_2$ holonomy. The conformal dimensions of the dual operators are in this case $\Delta = n + 2$ with $n = 0, 1, 2, \cdots$ for scalar fields and gauge field in the (n, n) representation. For type III case $n = 0$ is removed. For type I, it is given by $\Delta = n + 3$ in the $(n-1, n)$ representation and $\Delta = n + 1$ in the $(n+1, n)$ representation. Notice that the conformal dimensions are always integers. This is consistent with our identification of a D6-brane with a flavor for either of the two $U(N)$ gauge groups, where excitations of bi-fundamental scalars A_i and B_i should always include even number of these scalar fields with $\Delta = 1/2$ as explained before.

On the other hand, if we consider an open string between two different branes, then the fields coming from the open string receives the $\mathbb{Z}_2$ holonomy along the non-trivial cycle. In this case the conformal dimensions of dual operator are $\Delta = n + 5/2$ with $n = 0, 1, 2, \cdots$ for scalar fields and gauge field in the $(n + 1/2, n + 1/2)$ representation. For type I, it is given by $\Delta = n + 7/2$ in the $(n - 1/2, n + 1/2)$ representation and $\Delta = n + 3/2$ in the $(n + 3/2, n + 1/2)$ representation. The conformal dimensions take always half integer numbers in this case. Again these facts can be explained if we assume that two D6-branes with different $\mathbb{Z}_2$ Wilson lines correspond to flavor for two different gauge groups. In this way, our results of the spectrum support our identification of flavor D6-branes described in section 2.3.

4 Fractional Quantum Hall Effect and ABJM Theory

One interesting application of ABJM theory to condensed matter physics is to use it to model fractional quantum Hall effects holographically [3]. This stems from the fact that the low energy effective description of FQHE with filling faction $\nu = \frac{1}{k}$ (where $k \in \mathbb{Z}$) can be captured by a $U(1)_k$ Chern-Simons theory (see e.g. the text book [18]). The action is

$$S_{FQHE} = \frac{k}{4\pi} \int A \wedge dA + \frac{1}{2\pi} \int A \wedge F_{ext} \ , \tag{4.1}$$

where $F_{ext} = dA_{ext}$ is the external electromagnetic field applied to the FQHE sample, while A is the internal gauge field that describes the low energy degrees of freedom of FQHE.

In a FQHE system, the parity symmetry is broken. On the other hand, the original ABJM is parity-even since the two $U(N)$ gauge groups are interchanged during a parity transformation. Therefore, we need to break the parity symmetry of the ABJM theory in order to use it to model FQHE. One way to achieve this is by adding M fractional D2-branes (i.e. $D4$-branes wrapped on $\mathbb{CP}^1$). On the gravity side, these M fractional D2-branes are unstable and would fall into the horizon of AdS_4, leaving only NSNS 2-form flux behind [19]

$$\int_{\mathbb{CP}^1} B_2 = (2\pi)^2 \frac{M}{k} \ . \tag{4.2}$$

On the field theory side, the gauge group $U(N)_k \times U(N)_{-k}$ changes into $U(N+M)_k \times U(N)_{-k}$, thus breaking the parity symmetry. Treating the $U(N)_k \times U(N)_{-k}$ part which is common to both sides as spectators and extracting the $U(1) \subset U(M)$ part of the Chern-Simons gauge theory, we arrive at $U(1)_k$ Chern-Simons action (first term in (4.1)) that encodes the low energy description of FQHE.

Adding D4-brane wrapped on $\mathbb{CP}^1$ breaks the parity symmetry by shifting the rank of one of the two gauge groups. Another way to break parity symmetry is to add l D8-branes wrapped on $\mathbb{CP}^3$. As shown in [3, 20], it shifts the level of one of the gauge groups: $U(N)_k \times U(N)_{-k}$ changes into $U(N)_{k+l} \times U(N)_{-k}$. However, we will not discuss this system in the present work, leaving its application as a future problem.

In one of the models constructed in the recent paper [3], FQHE was realized by inserting defect D4 or D8-branes which are interpreted as edge states of the Chern-Simons gauge theory. Below we will show that we can also model FQHE without adding edges states, expressing everything purely in terms of RR-fluxes in the bulk AdS_4. Moreover, we will find that an addition of D6-branes enables us to describe a class of FQH plateau transitions.

4.1 Background RR-Field as External Field

The fractional D2-brane (namely D4-brane wrapped on $\mathbb{CP}^1$) has the world-volume action

$$S_{D4} = -T_4 \int d^5\sigma e^{-\phi}\sqrt{-\det(g+2\pi F)} + 2\pi^2 T_4 \int C_1 \wedge F \wedge F \ , \tag{4.3}$$

where $T_4 = (2\pi)^{-4}$ in the unit $\alpha' = 1$ and C_1 is sourced by the k units of the D6-brane flux, which leads to $\int_{\mathbb{CP}^1} F_2 = 2\pi k$. Integrating over the internal $\mathbb{CP}^1$, the Chern-Simons term of the D4-brane becomes

$$S^{CS}_{D4} = \frac{k}{4\pi}\int_{R^{1,2}} A \wedge dA \ . \tag{4.4}$$

Thus we obtained the first term in the Chern-Simons action (4.1) with the internal gauge field A. In other words, this Chern-Simons theory is the $U(M)$ part of the $U(N+M)\times U(N)$ gauge groups.

To couple the internal gauge field A to an external source A_{ext}, i.e. to realize the second term in action (4.1), we need to turn on some background RR-flux. Recall that the original ABJM theory has background RR-flux

$$F_2 = \frac{2k^2}{R^3}\omega \ , \quad \tilde{F}_4(\equiv F_4 - A_1 \wedge H_3) = -\frac{3}{8}R^3\epsilon_{AdS_4} \ , \quad H_3 = 0 \tag{4.5}$$

in $AdS_4 \times \mathbb{CP}^3$. We need to modify the RR-flux such that it provides an external gauge field that couple to A living on fractional D2-branes.

It is easy to see that we can simply turn on additional RR 3-form potential of the form

$$C_3 = \frac{4\pi k}{R^3} A_{ext} \wedge \omega \ , \tag{4.6}$$

where the 1-form A_{ext} lies inside AdS_4 and will serve as the external gauge field. This extra C_3 sources[9] an additional Chern-Simons term on the fractional D2-brane:

$$\frac{1}{(2\pi)^4}\int_{R^{1,2}\times\mathbb{CP}^1} 2\pi F \wedge C_3 = \frac{1}{2\pi}\int_{R^{1,2}} A_{ext} \wedge F \ , \tag{4.7}$$

which gives the correct coupling to the external field (i.e. the second term of (4.1)). Then after taking into account the kinetic term, we successfully realize the FQHE system coupled to the

[9] Note that the volume of $\mathbb{CP}^n$ is given by $\int_{\mathbb{CP}^n} \frac{\omega^n}{n!} = \frac{\pi^n}{n!}(2L)^{2n}$ in our convention.

background RR-field A_{ext} as defined by the action (4.1).[10]

A D-brane configuration modeling the FQHE has already been given in [21], which is constructed from D0, D2, D6 and D8-branes (see also [22] for other brane model of FQHE). This model looks similar to ours in the sense that a RR field plays the role of the external magnetic field in FQHE.[11] It would be very interesting to pursuit the relations further as this may lead to the holographic construction of [21].

4.2 Holographic Dual

Now we analyze the IIA supergravity with the modified RR-flux profile. We assume the following ansatz

$$F_2 = \frac{2k^2}{R^3}\omega + F_{D2} , \qquad \tilde{F}_4 = -\frac{3}{8}R^3\epsilon_{AdS_4} + \frac{4\pi k}{R^3}F_{ext}\wedge\omega , \tag{4.8}$$

and we require that the 3-form H_3 has indices only in the AdS_4 directions. Moreover, F_{D2} and F_{ext} have indices in the AdS_4 directions as well. We are interested in a combination of these fluxes which become a massless gauge field [1]. In this ansatz we find

$$*F_2 = \frac{R^3}{16}\epsilon_{AdS_4}\wedge\omega^2 + *_4F_{D2}\wedge\frac{\omega^3}{6} , \qquad *\tilde{F}_4 = \frac{k^2}{R^3}\omega^3 + \frac{2\pi k}{R^3} *_4 F_{ext}\wedge\omega^2 , \tag{4.9}$$

where $*$ and $*_4$ denote the Hodge duals in the total ten-dimensional spacetime and the AdS_4 spacetime, respectively. The equations of motion of fluxes are written at the linearized level as follows

$$\begin{gathered} dF_{D2} = 0 , \qquad dH_3 = 0 , \qquad dF_{ext} = -\frac{k}{2\pi}H_3 , \qquad d *_4 F_{D2} = \frac{6k^2}{R^3}H_3 , \\ d *_4 F_{ext} = 0 , \qquad \frac{1}{g_s^2} d *_4 H_3 = -\frac{24\pi k^3}{R^6} *_4 F_{ext} - \frac{6k^2}{R^3}F_{D2} . \end{gathered} \tag{4.10}$$

It is easy to see that the mode

$$F_{D2} = -\frac{4\pi k}{R^3} *_4 F_{ext} , \quad H_3 = 0 , \tag{4.11}$$

becomes a massless 2-form field strength. Under this constraint, the equations of motion (4.10) become exact even beyond the linear order approximation.

Now we assume that the background includes M fractional D2-branes, which correspond to the NSNS 2-form (4.2). If we concentrate on the massless mode (4.11), we find that the type IIA action is reduced to

$$S_{ext} = -\frac{R^3}{48\pi^2 k}\int_{AdS_4} F_{ext}\wedge *F_{ext} - \frac{M}{4\pi k}\int_{AdS_4} F_{ext}\wedge F_{ext} , \tag{4.12}$$

[10] Similarly, we can describe a FQHE system on the D2-branes instead of on fractional D2-branes by turning on some extra RR 1-form $C_1 \equiv A_{D2}$. The 1-from serves as the external gauge field, and it couples to the D2-brane in the standard way: $\frac{1}{2\pi}\int_{R^{1,2}} A_{D2}\wedge F$. In this paper we will only discuss FQHE system living on the fractional D2-branes.

[11] We would like to thank O. Bergman for pointing this out to us and for discussing possible relations.

where the second term comes from the Chern-Simons term of the IIA supergravity $-\frac{1}{4\kappa^2}\int B_2 \wedge dC_3 \wedge dC_3$. In the second term of the action, the topological term $\int F_{ext} \wedge F_{ext}$ leads to a boundary Chern-Simons term $\int A_{ext} \wedge F_{ext}$ in the AdS/CFT procedure (see also [23]). Since A_{ext} is the external gauge field probing FQHE, we can immediately read off the fractionally quantized Hall conductivity:

$$\sigma_{xy} = \frac{M}{2\pi k} = \frac{M}{k} \cdot \frac{e^2}{h} \ , \tag{4.13}$$

where we have restored the electron charge e and $\hbar = 1$.

In this way we have shown that the ABJM theory with $M > 0$ fractional D2-branes can model fractional quantum Hall effect. Something interesting also happens at $M = 0$. If we focus on the gauge theory part while ignoring the gravity part, the theory as given by action (4.12) with $M = 0$ has an S-duality that inverts the Yang-Mills coupling, as was also noted in a different example [24]. Since the coupling is given by $g_{YM} = \sqrt{\frac{12\pi^2 k}{R^3}} \sim (\frac{k}{N})^{1/4}$, this "S-duality" exchanges the level k and the rank N in the ABJM theory.

4.3 Flavor D6-branes and Quantum Hall Transition

In the previous system the Hall conductivity is fractionally quantized, and the system describes one plateau of FQHE. In order to describe the plateau transition in FQHE, where σ_{xy} changes continuously, we would like to add flavor D6-branes in this setup and consider its deformation. Though the mechanism in our transition described below is very similar to the one in D3-D7 systems discussed in [25], its interpretation is different. This is because in our case we regard the RR field A_{ext} as the external gauge field, whereas in [25] the external gauge field is given by the gauge field on the D7-brane. This is the main reason why we can realize the plateau-transition for the fractional QHE, while the paper [25] realized the transition in the integer QHE.

In the calculation of σ_{xy}, new contributions essentially come from the Chern-Simons terms of the D6-branes. We consider a D6-branes wrapped on $S^3/\mathbb{Z}_2$ (2.13) in the presence of NSNS B-field (4.2). Its non-trivial Chern-Simons terms are

$$S_{D6-RR} = \frac{1}{(2\pi)^5}\int C_3 \wedge F \wedge B_{NS} + \frac{1}{2(2\pi)^4}\int C_1 \wedge B_{NS} \wedge F \wedge F \ . \tag{4.14}$$

Plugging in the explicit forms of B_{NS} and C_3, we find

$$S_{D6-RR} = \frac{M}{2\pi k}\zeta \int_{R^{1,2}} A_{ext} \wedge F + \frac{M}{4\pi}\zeta \int_{R^{1,2}} A \wedge F \ , \tag{4.15}$$

where A is the $U(1)$ gauge field on the D6-brane and A_{ext} is the external field induced by the RR 3-from potential. The quantity ζ is defined by

$$\int_{[D6]} \omega \wedge \omega = 16\pi^2 L^4 \zeta \ , \tag{4.16}$$

where $[D6]$ is the four-dimensional worldvolume of the D6-branes in the $\mathbb{CP}^3$ directions. Here we normalized such that $\zeta = 1/2$ when the D6-brane wraps the following 4-cycle

$$0 \le \xi < \frac{\pi}{4}\ ,\quad 0 \le \psi < 2\pi\ ,\quad 0 \le \theta < \pi\ ,\quad 0 \le \varphi < 2\pi\ . \tag{4.17}$$

After combining S_{D6-RR} with the boundary Chern-Simons term coming from the topological term in (4.12) and classically integrating out A, we finally obtain

$$S_{tot-RR} = \frac{M}{4\pi}\left(\frac{1}{k} - \frac{\zeta}{k^2}\right)\int_{R^{1,2}} A_{ext} \wedge F_{ext}\ . \tag{4.18}$$

Now we put the system at the finite temperature. The AdS_4 is then replaced by a black hole solution. We can find (at least numerically) solutions whose embedding function $\xi(r)$ changes smoothly from $\xi(\infty) = \frac{\pi}{4}$ to $\xi(r_0) = 0$ for certain large enough r_0. Notice that the point $\xi = 0$ is interpreted in the IIB brane configuration as where the flavor D5-brane and the $(1,k)$5-brane make a $\mathcal{N} = 3$ supersymmetric bound-state i.e. the $(1,k+1)$5-brane as is seen from (2.23). A large value of r_0 corresponds to large mass of the hypermultiplets. For flavor masses large enough, the D6-brane does not touch the horizon. As we reduce r_0 (or equivalently the flavor mass), the D6-brane will move closer to the horizon and only stay away from it above some critical value of r_0. Then if we reduce the flavor mass further, the D6-brane will terminate at the horizon as is known in the D3-D7 system [26]. When this happens, the value of ζ jumps from $\zeta = 1/2$ (for the D6-brane separated from the horizon) to a certain value $\zeta = \zeta_0 < 1/2$, as in the D3-D7 system of [25]. As the flavor mass becomes smaller, ζ gets smaller and finally reaches $\zeta = 0$, which corresponds to the original flavor D6-brane. This describes half of the transition process, and the other half can be found similarly. Therefore, as we change the flavor mass, the dual FQHE system undergoes a plateau transition from $\nu = \frac{1}{k}$ to $\nu = \frac{1}{k+1}$ (recall that we assumed that k is large in (4.18) so that the description of type IIA superstring theory is reliable). If we combine two D6-branes, then we can realize more realistic transitions from $\nu = \frac{1}{k}$ to $\nu = \frac{1}{k+2}$.[12] It will be an interesting future problem to examine the above transition in more detail and calculate how σ_{xx} and σ_{xy} change explicitly.

Finally notice that in the CFT side, this shift of the level can be understood as the parity anomaly by adding mass to the hypermultiplets and integrating them out. As mentioned in [13] there are three supersymmetric and one non-supersymmetric mass deformations. The former correspond to the shifts of $\vec{x}_2$ in the IIB brane configuration and appear in the fluctuation spectrum of scalar modes in section 3. The parity anomaly only occurs in the latter one. Therefore we expect our D6-brane configuration assumed in this subsection to be non-supersymmetric.

[12] The elementary particles in realistic models are either purely fermions as in traditional 2DEG FQHE systems or purely bosons as in the more recent bosonic FQHE in rotating cold atoms. Thus during a plateau transition k jumps only among odd integers if in fermionic systems, and only among even integers if in bosonic ones.

5 Conclusion

In this paper, we have performed an analysis on probe branes dual to the flavors in the $\mathcal{N}=6$ Chern-Simons theory, i.e., the ABJM theory. We found that the probe branes are wrapped over $AdS_4 \times S^3/\mathbb{Z}_2$ in the dual geometry of $AdS_4 \times \mathbb{CP}^3$. These are classified into two types by the $\mathbb{Z}_2$ Wilson line and each corresponds to a flavor for each of the two $U(N)$ gauge groups. The probe D6-brane is shown to preserve 12 supersymmetries, which are the same supersymmeties of the dual $\mathcal{N}=3$ superconformal symmetry. The brane configuration is also confirmed by the analysis of a type IIB brane configuration dual to the ABJM theory with flavors. We obtained the spectrum of BPS mesonic operators in the ABJM theory with flavors by analyzing the fluctuations of the dual D6-branes and found agreements with our expectation. We also considered an application of the $\mathcal{N}=6$ Chern-Simons theory to the fractional quantum Hall effect. In the presence of fractional D2-branes, we showed that it offers us a simple holographic setup of fractional quantum Hall effect. Moreover, we found that mass deformations of the flavor D6-brane are interpreted as plateau transitions of fractional quantum Hall effect.

There are several future directions we would like to consider. First of all, we have only analyzed the flavor branes preserving the maximal supersymmetry, and it would be important to look for branes preserving less supersymmetry. Moreover, there are other supersymmetric Chern-Simons theories with holographic duals such as the one with the orientifold discussed in [19]. Therefore it is possible to extend our analysis of flavor D-branes to these cases. For the purpose of application to condensed matter physics, it is very interesting to explicitly compute the conductivities in our setup at finite temperatures. Furthermore, the relation to the description of the plateau transition using the Chern-Simons Ginzburg-Landau model [27] should also be clarified.

Note added: While we were preparing the draft, we noticed that the paper [28] appeared in the arXiv. It has a major overlap on the discussions of D6-branes as the holographic dual of adding flavors in the $\mathcal{N}=6$ Chern-Simons theory, though the identification is slightly different. After our paper appeared on the arXiv, we found the paper [29] listed on the same day, which also argued the same interpretation on the flavor D6-branes as ours.

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A Supersymmetry and Brane Configuration

Here we count the number of supersymmetries preserved by the type IIB brane configuration system discussed in section 2.5. It is convenient to combine the two chiral spinors (16 components each) ϵ_L and ϵ_R in the type IIB supergravity by a complex 16 component spinor as $\epsilon = \epsilon_L + i\epsilon_R$. In this combination it satisfies the chirality condition as

$$\gamma_{0123456789}\epsilon = \epsilon \ . \tag{A.1}$$

Let us study which kind of supersymmetry is preserved in the presence of branes. For a D3-brane in the 0126-directions, preserved supersymmetry is associated with the spinor satisfying

$$\epsilon = -i\gamma_{0126}\epsilon \ . \tag{A.2}$$

Similarly we have the constraint

$$\epsilon = i\gamma_{012345}\bar{\epsilon} \ , \tag{A.3}$$

for a D5-brane in the 012345-directions and

$$\epsilon = \gamma_{012789}\bar{\epsilon} \ . \tag{A.4}$$

for a NS5-brane in the 012789-directions.

First we show that this D3-D5-NS5 system preserves 1/4 of the 32 supersymmetries. Indeed, we can derive one of the three conditions (A.2), (A.3) and (A.4) from the other two. Diagonalizing the spinor by the actions of γ_{37}, γ_{48} and γ_{59} as

$$\gamma_{37}\epsilon = is_1\epsilon \ , \qquad \gamma_{48}\epsilon = is_2\epsilon \ , \qquad \gamma_{59}\epsilon = is_3\epsilon \ , \tag{A.5}$$

the chirality constraint (A.1) leads to

$$\gamma_{0126}\epsilon = i(s_1s_2s_3)\epsilon \ , \qquad \gamma_{0126}\bar{\epsilon} = -i(s_1s_2s_3)\bar{\epsilon} \ . \tag{A.6}$$

The degrees of freedom of spinor is specified by (s_1, s_2, s_3) and this leads to 8 complex components. Moreover, the condition (A.2) requires

$$s_1s_2s_3 = 1 \ , \tag{A.7}$$

and thus we have the following 4 possibilities

$$(s_1, s_2, s_3) = (+++), (--+), (-+-), (+--) \ . \tag{A.8}$$

In this way the D3-D5-NS5 system preserves 1/4 of the 32 supersymmetries due to the constraints (A.4) and (A.7).

In order to construct the brane configuration in section 2.5, we further insert a $(1, k)$5-brane. We assumed that it is rotated by the same angle θ in each of 37, 48 and 59 planes. If we

choose that this angle θ is given by $\sin\theta = k/\sqrt{1+k^2}$ (assuming $g_s = 1$ and vanishing axion for simplicity), then we find the following supersymmetry constraint on the spinor as

$$\epsilon = e^{i\theta} \cdot \gamma_{012789} \cdot e^{-\theta(\gamma_{37}+\gamma_{48}+\gamma_{59})}\ \bar{\epsilon}\ . \tag{A.9}$$

Suppose that $\epsilon_{(0)}$ satisfies the supersymmetric condition for the D3-D5-NS5 system. Then, with the help of (A.4), we can find that the following spinor ϵ satisfies the condition (A.9) as

$$\epsilon = e^{i\frac{\theta}{2}+\frac{\theta}{2}(\gamma_{37}+\gamma_{48}+\gamma_{59})}\epsilon_{(0)}\ . \tag{A.10}$$

Since the $\epsilon_{(0)}$ are the spinors for the original system, we would like to find spinors that satisfies $\epsilon = \epsilon_{(0)}$. They correspond to supersymmetries surviving after adding a $(1,k)$5-brane. This is given by the choice $(--+), (-+-), (+--)$ in the (A.8). Thus we have found that the $(1,k)$5-brane further breaks 1/4 out of the original 8 supersymmetries of D3-D5-NS5 system. In this way, we have shown that this final system preserves 6 supersymmetries corresponding to $\mathcal{N} = 3$ Chern-Simons theory with flavors.

B The Orbifold Model on $S^3/\mathbb{Z}_2$

In this appendix we analyze which modes of spherical harmonics survive under the $\mathbb{Z}_2$ orbifold projection. The symmetry of S^3 is $SO(4) \sim SU(2)_R \times SU(2)_L$ and the function can be labeled by the representation of $SU(2)_R \times SU(2)_L$. We denote $(m, \bar{m})$ as the eigenfunction of J^3_R, J^3_L. For the scalar function in the $(\frac{l}{2}, \frac{l}{2})$ representation, the both run the same range as $m, \bar{m} = -l/2, -l/2+1, \cdots, l/2$. The scalar function satisfies

$$D_i D^i Y^l(S^3) = -l(l+2)Y^l(S^3)\ , \tag{B.1}$$

where D_i are the covariant derivatives on S^3. In our case, the identification is taken for the shift $\psi \to \psi + 4\pi/p$ with $p = 2$, and the corresponding orbifold model is obtained by the projection operator (see, e.g., [30, 31])

$$P = \tfrac{1}{2}(1 + e^{4\pi i J^3_L/2})\ . \tag{B.2}$$

Therefore, only $2\bar{m} \in 2\mathbb{Z}$ survives under the orbifold projection, which implies $l \in 2\mathbb{Z}$ for the scalar function. The vector spherical harmonics are in the $(\frac{l\pm1}{2}, \frac{l\mp1}{2})$ representation, and satisfy

$$D_i D^i Y^{l,\pm}_j(S^3) - R^k{}_j Y^{l,\pm}_k(S^3) = -(l+1)^2 Y^{l,\pm}_j(S^3) \tag{B.3}$$

with $R^k{}_j$ as the Ricci tensor of S^3 . The projection is the same as in the scalar case, thus the restriction is $2\bar{m} \in 2\mathbb{Z}$, which implies $l \in 2\mathbb{Z}+1$.

Let us consider the effects of $\mathbb{Z}_2$ Wilson loop along the non-trivial cycle. We prepare two fractional branes with and without $\mathbb{Z}_2$ Wilson loop. The gauge symmetry is now $U(1) \times U(1)$,

and there are two types of open string between the same brane and between different branes. The fields coming from the former do no feel any effect of Wilson loop and the projection is the same as before. For the other fields coming from the latter, the projection becomes

$$P = \frac{1}{2}(1 - e^{4\pi i J_L^3/2}) \tag{B.4}$$

due to the existence of Wilson loop. For this type of scalar field the restriction is $2\bar{m} \in 2\mathbb{Z}+1$, which exists for $l \in 2\mathbb{Z}+1$. For this type of vector field the restriction leads to $l \in 2\mathbb{Z}$.

References

[1] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, "$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," JHEP **0810** (2008) 091 [arXiv:0806.1218 [hep-th]].

[2] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200];

[3] M. Fujita, W. Li, S. Ryu and T. Takayanagi, "Fractional quantum Hall effect via holography: Chern-Simons, edge states, and hierarchy," arXiv:0901.0924 [hep-th].

[4] Y. Nakayama, S. Ryu, M. Sakaguchi and K. Yoshida, "A family of super Schrödinger invariant Chern-Simons matter systems," JHEP **0901** (2009) 006 [arXiv:0811.2461 [hep-th]]; Y. Nakayama, M. Sakaguchi and K. Yoshida, "Non-relativistic M2-brane gauge theory and new superconformal algebra," arXiv:0902.2204 [hep-th].

[5] A. Karch and E. Katz, "Adding flavor to AdS/CFT," JHEP **0206**, 043 (2002) [arXiv:hep-th/0205236].

[6] D. Gaiotto and X. Yin, "Notes on superconformal Chern-Simons-matter theories," JHEP **0708** (2007) 056 [arXiv:0704.3740 [hep-th]].

[7] J. Polchinski, "String theory. Vol. 1,2," *Cambridge, UK: Univ. Pr.* (1998).

[8] M. Benna, I. Klebanov, T. Klose and M. Smedback, "Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence," JHEP **0809** (2008) 072 [arXiv:0806.1519 [hep-th]].

[9] T. Nishioka and T. Takayanagi, "On type IIA Penrose limit and $\mathcal{N} = 6$ Chern-Simons theories," JHEP **0808** (2008) 001 [arXiv:0806.3391 [hep-th]].

[10] M. R. Douglas and G. W. Moore, "D-branes, quivers, and ALE instantons," arXiv:hep-th/9603167.

[11] T. Nishioka and T. Takayanagi, "Fuzzy Ring from M2-brane Giant Torus," JHEP **0810** (2008) 082 [arXiv:0808.2691 [hep-th]].

[12] T. Kitao, K. Ohta and N. Ohta, "Three-dimensional gauge dynamics from brane configurations with (p,q)-fivebrane," Nucl. Phys. B **539**, 79 (1999) [arXiv:hep-th/9808111].

[13] O. Bergman, A. Hanany, A. Karch and B. Kol, "Branes and supersymmetry breaking in 3D gauge theories," JHEP **9910**, 036 (1999) [arXiv:hep-th/9908075].

[14] V. Niarchos, "R-charges, chiral rings and RG flows in supersymmetric Chern-Simons-matter theoreis," arXiv:0903.0435 [hep-th].

[15] A. Hanany and E. Witten, "Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics," Nucl. Phys. B **492**, 152 (1997) [arXiv:hep-th/9611230].

[16] J. P. Gauntlett, G. W. Gibbons, G. Papadopoulos and P. K. Townsend, "Hyper-Kähler manifolds and multiply intersecting branes," Nucl. Phys. B **500**, 133 (1997) [arXiv:hep-th/9702202].

[17] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, "Meson spectroscopy in AdS/CFT with flavour," JHEP **0307**, 049 (2003) [arXiv:hep-th/0304032].

[18] X-G. Wen, "Quantum field theory of Many-Body systems," *Oxford University Press* (2004).

[19] O. Aharony, O. Bergman and D. L. Jafferis, "Fractional M2-branes," JHEP **0811** (2008) 043 [arXiv:0807.4924 [hep-th]].

[20] D. Gaiotto and A. Tomasiello, "The gauge dual of Romans mass," arXiv:0901.0969 [hep-th].

[21] O. Bergman, Y. Okawa and J. H. Brodie, "The stringy quantum Hall fluid," JHEP **0111** (2001) 019 [arXiv:hep-th/0107178]; O. Bergman, "Quantum Hall physics in string theory," arXiv:hep-th/0401106.

[22] J. H. Brodie, L. Susskind and N. Toumbas, "How Bob Laughlin tamed the giant graviton from Taub-NUT space," JHEP **0102**, 003 (2001) [arXiv:hep-th/0010105].

[23] E. Keski-Vakkuri and P. Kraus, "Quantum Hall effect in AdS/CFT," JHEP **0809** (2008) 130 [arXiv:0805.4643 [hep-th]].

[24] C. P. Herzog, P. Kovtun, S. Sachdev and D. T. Son, "Quantum critical transport, duality, and M-theory," Phys. Rev. D **75**, 085020 (2007) [arXiv:hep-th/0701036].

[25] J. L. Davis, P. Kraus and A. Shah, "Gravity dual of a quantum Hall plateau transition," JHEP **0811** (2008) 020 [arXiv:0809.1876 [hep-th]].

[26] D. Mateos, R. C. Myers and R. M. Thomson, "Holographic phase transitions with fundamental matter," Phys. Rev. Lett. **97** (2006) 091601 [arXiv:hep-th/0605046].

[27] S. Kivelson, D-H. Lee, S-C. Zhang, "Global phase diagram in the quantum Hall effect," Phys. Rev. B **46** (1992) 2223.

[28] S. Hohenegger and I. Kirsch, "A note on the holography of Chern-Simons matter theories with flavour," arXiv:0903.1730.

[29] D. Gaiotto and D. L. Jafferis, "Notes on adding D6 branes wrapping $\mathbb{RP}^3$ in $AdS_4 \times \mathbb{CP}^3$," arXiv:0903.2175 [hep-th].

[30] H. Lin and J. M. Maldacena, "Fivebranes from gauge theory," Phys. Rev. D **74**, 084014 (2006) [arXiv:hep-th/0509235].

[31] Y. Hikida, "Phase transitions of large N orbifold gauge theories," JHEP **0612**, 042 (2006) [arXiv:hep-th/0610119].